Convolutions of radial exponential densities

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I will show new estimates for convolutions and densities of compound Poisson measures (denoted by p_λ), of multivariate, radial densities. Previous methods were limited to the situation where the second convolution of densities is comparable at infinity to the initial density. We propose a new approach which allows us to break down this barrier.

The first main result gives the upper estimate for convolutions.

Theorem 1.

$$\frac{f^{n\star}(x)}{f(x)} \leq \sum_{i=1}^{n} \binom{n}{i} M^{n-i} h_i(x), \quad x \in \mathbb{R}^d,\ n \in \mathbb{N},$$

where M is explicit constant and h_n's are auxiliary functions that arise from $\frac{f^{n\star}(x)}{f(x)}$ by reducing domain of integration.

The corresponding lower estimate which is sufficient for applications is more direct.

In the second part of the talk i will concentrate on the specific class of densities $f(x) = e^{-m|x|}|x|^{-\gamma}$ where $m > 0, \gamma \in [0, \frac{d+1}{2}]$. For such functions, we obtain the following explicit estimates.

Theorem 2.

$$D_1^{n-1} \frac{\Gamma(\rho_1)^n}{\Gamma(\rho_1 n)} \leq \frac{f^{n\star}(x)}{f(x)|x|^{\left(\frac{d+1}{2}-\gamma\right)(n-1)}} \leq D_2^{n-1} \frac{\Gamma(\rho_2)^n}{\Gamma(\rho_2 n)} + O\left(\frac{1}{|x|^{\frac{d+1}{2}-\gamma}}\right).$$

where D_1, D_2, ρ_1, ρ_2 are explicit constants.

Theorem 3.

$$\phi\big(\rho_1, 0; \kappa_1 \lambda |x|^{\frac{d+1}{2}-\gamma}\big) \leq \frac{p_\lambda(x)}{e^{-\lambda\|f\|_1} e^{-m|x|} |x|^{-\frac{d+1}{2}}} \leq e^{M_2\lambda} \phi\big(\rho_2, 0; \kappa_2 \lambda |x|^{\frac{d+1}{2}-\gamma}\big)$$

where ϕ is generalized Bessel function [1] *and $\rho_1, \rho_2, \kappa_1, \kappa_2$ are explicit constants.*

The results in last theorem are further investigated by using asymptotics of the generalized Bessel function [1]. The talk is based on joint paper with K. Kaleta [2].

References

[1] E. Maitland Wright, *The Asymptotic Expansion of the Generalized Bessel Function.* Proceedings of the London Mathematical Society. s2-38(1):257-270, 01 1935.

[2] M. Baraniewicz, K. Kaleta, *Exponential Densities and compound Poisson measures.* ArXiv2206.02258. 2022+.