Convolutions of radial exponential densities
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I will show new estimates for convolutions and densities of compound Poisson measures (denoted by \( p_\lambda \)), of multivariate, radial densities. Previous methods were limited to the situation where the second convolution of densities is comparable at infinity to the initial density. We propose a new approach which allows us to break down this barrier.

The first main result gives the upper estimate for convolutions.

**Theorem 1.**

\[
f_{n}^{*}(x) \leq \sum_{i=1}^{n} \binom{n}{i} M^{n-i} h_i(x), \quad x \in \mathbb{R}^d, \quad n \in \mathbb{N},
\]
where \( M \) is explicit constant and \( h_i \)'s are auxiliary functions that arise from \( \frac{f_{n}^{*}(x)}{f(x)} \) by reducing domain of integration.

The corresponding lower estimate which is sufficient for applications is more direct.

In the second part of the talk I will concentrate on the specific class of densities

\[ f(x) = e^{-m|x|}|x|^{-\gamma} \]
where \( m > 0, \gamma \in [0, \frac{d+1}{2}] \). For such functions, we obtain the following explicit estimates.

**Theorem 2.**

\[
D_1^{n-1} \frac{\Gamma(\rho_1)^n}{\Gamma(\rho_1 n)} \leq \frac{f_{n}^{*}(x)}{f(x)|x|^{\left(\frac{d+1}{2} - \gamma\right)(n-1)}} \leq D_2^{n-1} \frac{\Gamma(\rho_2)^n}{\Gamma(\rho_2 n)} + O \left( \frac{1}{|x|^{\frac{d+1}{2} - \gamma}} \right),
\]
where \( D_1, D_2, \rho_1, \rho_2 \) are explicit constants.

**Theorem 3.**

\[
\phi(\rho_1, 0; \kappa_1 \lambda |x|^{\frac{d+1}{2} - \gamma}) \leq \frac{p_\lambda(x)}{e^{-\lambda \|f\|_1} e^{-m|x|} |x|^{-\frac{d+1}{2}}} \leq e^{M_2 \lambda} \phi(\rho_2, 0; \kappa_2 \lambda |x|^{\frac{d+1}{2} - \gamma})
\]
where \( \phi \) is generalized Bessel function [1] and \( p_1, p_2, \kappa_1, \kappa_2 \) are explicit constants.

The results in last theorem are further investigated by using asymptotics of the generalized Bessel function [1]. The talk is based on joint paper with K. Kaleta [2].

**References**
